OCR
RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS

4723 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

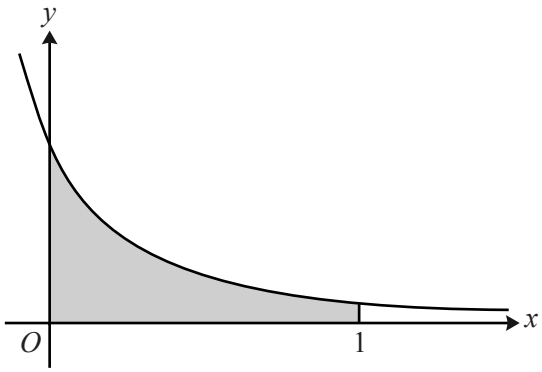
This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Show that $\int_{\sqrt{2}}^{\sqrt{6}} \frac{2}{x}\,\mathrm{d}x = \ln 3$. **[3]**

2



The diagram shows part of the curve $y = \frac{6}{(2x+1)^2}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 1$ and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis. **[5]**

3 Find the equation of the normal to the curve $y = \frac{x^2+4}{x+2}$ at the point $(1, \frac{5}{3})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

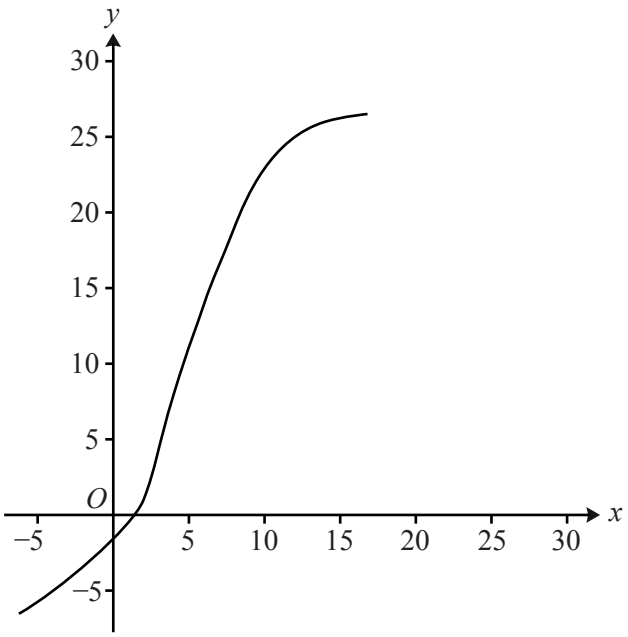
4 The acute angles α and β are such that

$$2\cot\alpha = 1 \quad \text{and} \quad 24 + \sec^2\beta = 10\tan\beta.$$

(i) State the value of $\tan\alpha$ and determine the value of $\tan\beta$. **[4]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

5



It is given that f is a one-one function defined for all real values. The diagram shows the curve with equation $y = \mathrm{f}(x)$. The coordinates of certain points on the curve are shown in the following table.

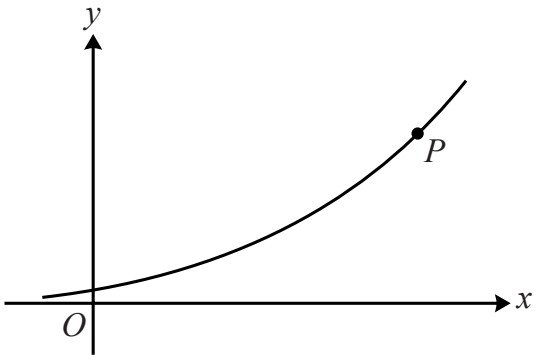
x	2	4	6	8	10	12	14
y	1	8	14	19	23	25	26

(i) State the value of $\mathrm{ff}(6)$ and the value of $\mathrm{f}^{-1}(8)$. **[2]**

(ii) On the copy of the diagram, sketch the curve $y = \mathrm{f}^{-1}(x)$, indicating how the curves $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related. **[2]**

(iii) Use Simpson's rule with 6 strips to find an approximation to $\int_2^{14} \mathrm{f}(x)\,\mathrm{d}x$. **[4]**

6



The diagram shows the curve with equation $x = \ln(y^3 + 2y)$. At the point P on the curve, the gradient is 4 and it is given that P is close to the point with coordinates (7.5, 12).

(i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **[2]**

(ii) Show that the y-coordinate of P satisfies the equation

$$y = \frac{12y^2 + 8}{y^2 + 2}.$$ **[3]**

(iii) By first using an iterative process based on the equation in part **(ii)**, find the coordinates of P, giving each coordinate correct to 3 decimal places. **[5]**

7 **(i)** Substance A is decaying exponentially and its mass is recorded at regular intervals. At time t years, the mass, M grams, of substance A is given by

$$M = 40\mathrm{e}^{-0.132t}.$$

(a) Find the time taken for the mass of substance A to decrease to 25% of its value when $t = 0$. **[3]**

(b) Find the rate at which the mass of substance A is decreasing when $t = 5$. **[3]**

(ii) Substance B is also decaying exponentially. Initially its mass was 40 grams and, two years later, its mass is 31.4 grams. Find the mass of substance B after a further year. **[3]**

8 **(i)** Express $\cos 4\theta$ in terms of $\sin 2\theta$ and hence show that $\cos 4\theta$ can be expressed in the form $1 - k\sin^2\theta\cos^2\theta$, where k is a constant to be determined. **[3]**

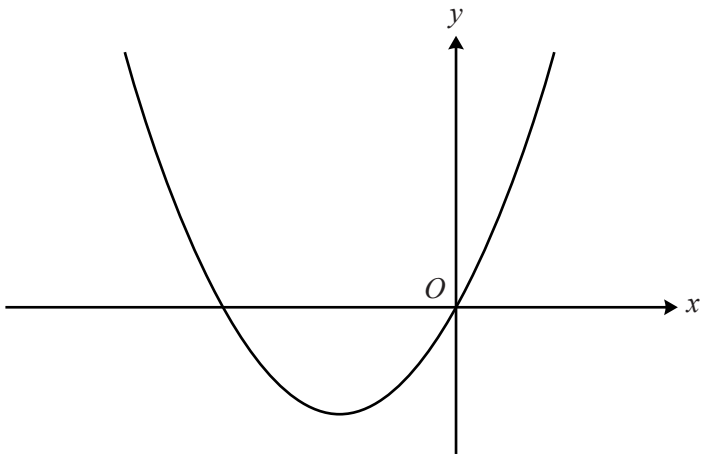
(ii) Hence find the exact value of $\sin^2(\frac{1}{24}\pi)\cos^2(\frac{1}{24}\pi)$. **[2]**

(iii) By expressing $2\cos^2 2\theta - \frac{8}{3}\sin^2\theta\cos^2\theta$ in terms of $\cos 4\theta$, find the greatest and least possible values of

$$2\cos^2 2\theta - \tfrac{8}{3}\sin^2\theta\cos^2\theta$$

as θ varies. **[5]**

9



The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x),$$

where k is a positive constant. The diagram shows the curve with equation $y = f(x)$.

(i) The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations:
a translation parallel to the x-axis,
a translation parallel to the y-axis,
a stretch.
Give details, in terms of k where appropriate, of these transformations. **[5]**

(ii) Find the range of f in terms of k. **[2]**

(iii) It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case. **[6]**

Question			Answer	Marks	Guidance	
1			State $2\ln x$ Use both relevant logarithm properties correctly Obtain $\ln 3$	B1 M1 A1 **[3]**	may be implied by immediate use of limits either or both may be implied, eg by $2\ln\sqrt{6} = \ln 6$ or by $\ln 6 - \ln 2 = \ln 3$ AG; with at least one property shown explicitly	
2			State volume is $\int \frac{36\pi}{(2x+1)^4}\,\mathrm{d}x$ Obtain integral of form $k(2x+1)^n$ Obtain $-6\pi(2x+1)^{-3}$ or $-6(2x+1)^{-3}$ Substitute correct limits and subtract Obtain $\frac{52}{9}\pi$	B1 M1 A1 M1 A1 **[5]**	or equiv in terms of x; no need for limits; condone absence of dx; condone absence of π here if it appears later in solution (even as part of a wrong answer) for any $n \leq -1$; with or without π; or ku^n following substitution; allow if $n = -5$; allow M1 if one slight slip occurs in $(2x+1)$ or (unsimplified) equiv the correct way round for integral of form $k(2x+1)^{-3}$; allow if one slight slip occurs in $(2x+1)$; not earned if limit 0 leads to $\ldots - 0$ or similarly simplified exact equiv	

Question			Answer	Marks	Guidance	
3			Attempt use of quotient rule	M1	condone u/v muddles but needs $(x+2)^2$ in denominator; condone numerator back to front; or product rule to produce terms involving $(x+2)^{-1}$ and $(x+2)^{-2}$	
			Obtain $\frac{2x(x+2)-(x^2+4)}{(x+2)^2}$	A1	or equiv; brackets may be implied by subsequent recovery	
			Substitute 1 into attempt at first derivative	M1	also allow if sign slip leads to derivative cancelling to 1	
			Obtain $\frac{1}{9}$	A1		
			Use -9 as gradient of normal	A1ft	following their value of first derivative	
			Attempt to find equation of normal	M1	not equation of tangent; needs use of negative reciprocal of their derivative value	
			Obtain $27x+3y-32=0$	A1	or equiv of requested form	
				[7]		
4	(i)		State $\tan\alpha = 2$	B1	ignoring subsequent work to find angle	
			Use identity $\sec^2\beta = 1+\tan^2\beta$	B1		
			Attempt solution of quad eqn for $\tan\beta$	M1	3 term quad eqn; using reasonable attempt at factorisation to find value or use of quadratic formula (with no more than one slip)	
			Obtain $\tan\beta = 5$	A1	ignoring subsequent work to find angle; value 5 must be obtained legitimately	
				[4]		

Question			Answer	Marks	Guidance	
4	(ii)		Substitute their values of $\tan\alpha$ and $\tan\beta$ in formula … Obtain $\frac{2+5}{1-2\times5}$ Obtain $-\frac{7}{9}$	M1 A1ft A1 **[3]**	… of form $\frac{\pm\tan\alpha\pm\tan\beta}{\pm1\pm\tan\alpha\tan\beta}$ following their values from part **(i)** or correct simplified exact equiv including $\frac{7}{-9}$; A0 if $\tan\beta=5$ obtained incorrectly in part **(i)** SC: use of calculator for $\tan(\tan^{-1}2+\tan^{-1}5)$ to give $-\frac{7}{9}$ earns all 3 marks (but 0 out of 3 if answer is not exact); with either or both of 2 and 5 wrong, 2 out of 3 available for this approach if result is exact and correct given their two values	
5	(i)		State 26 State 4	B1 B1 **[2]**		
5	(ii)		Sketch (more or less) correct curve Refer to reflection in $y=x$ or symmetrical about $y=x$ or mirrored in $y=x$	B1 B1 **[2]**	with approx correct curvatures and curve going through second quadrant but not fourth quadrant; allow if sketch does not meet given curve on line $y=x$ explicit reference needed, not just line $y=x$ shown on sketch	

Question			Answer	Marks	Guidance	
5	(iii)		Attempt calculation $k(y+4y+2y+...)$	M1	any constant k; with y-values from table and coefficients 1, 2 and 4 occurring at least once each; brackets may be implied by subsequent calculation	
			Obtain $k(1+32+28+76+46+100+26)$	A1	or (unsimplified) equiv	
			Use $k=\frac{1}{3}\times 2$	A1		
			Obtain 206	A1 **[4]**		
6	(i)		Obtain rational expression of form $\frac{f(y)}{y^3+2y}$	M1	where $f(y)$ is not constant; ignore how expression is labelled	
			Obtain $\frac{3y^2+2}{y^3+2y}$	A1 **[2]**		
6	(ii)		Recognise that $\frac{dy}{dx}=1\div\frac{dx}{dy}$ for rational expression of form $\frac{f(y)}{y^3+2y}$	M1	may be implied	
			Obtain $\frac{y^3+2y}{3y^2+2}=4$ or $\frac{3y^2+2}{y^3+2y}=\frac{1}{4}$	A1ft	following their rational expression from **(i)**	
			Confirm $y=\frac{12y^2+8}{y^2+2}$	A1 **[3]**	AG; following correct work and with at least one step between $\frac{y^3+2y}{3y^2+2}=4$ or equiv and answer	

Question			Answer	Marks	Guidance	
6	(iii)		Obtain correct first iterate 11.89	B1	or greater accuracy; having started with 12; accept if 12 used in part **(ii)** to produce next value and 11.89 used as starting value here	
			Attempt iteration process to produce at least 3 iterates in all	M1	implied by plausible sequence of values; having started anywhere; if formula clearly not based on equation from part **(ii)**, award M0	
			Obtain at least 2 more correct iterates	A1	showing at least 3 decimal places	
			Obtain 11.888 for y	A1	answer needed to exactly 3 decimal places	
			Obtain 7.441 for x	A1	answer needed to exactly 3 decimal places; award final A0 if not clear which is x and which is y	
					[12 → 11.89041 → 11.88841 → 11.88837]	
				[5]		

Question			Answer	Marks	Guidance	
7	(i)	(a)	State or imply $e^{-0.132t} = 0.25$ Attempt solution of eqn of form $e^{-0.132t} = k$ Obtain 10.5	B1 M1 A1 **[3]**	or equiv such as $40e^{-0.132t} = 10$ using sound process; implied by correct ans; allow trial and improvement attempt or greater accuracy	
7	(i)	(b)	Differentiate to obtain $ke^{-0.132t}$ Obtain $5.28e^{-0.132t}$ or $-5.28e^{-0.132t}$ Substitute 5 to obtain 2.73 or –2.73	M1 A1 A1 **[3]**	where k is a constant not equal to 40 (allow even if process looks like integration) or (unsimplified) equiv accept 2.7 or –2.7 or greater accuracy; allow 2.73 or –2.73 whatever it is claimed to be	
7	(ii)		EITHER Attempt to solve $40e^{2\lambda} = 31.4$ or $40e^{-2\lambda} = 31.4$ Obtain or imply $40e^{-0.121t}$ Substitute 3 to obtain 27.8 OR Attempt calculation involving multiplication of power of $\frac{31.4}{40}$ Obtain $31.4\times(\frac{31.4}{40})^{0.5}$ or $40\times(\frac{31.4}{40})^{1.5}$ Obtain 27.8	 M1 A1 A1 **[3]** M1 A1 A1	 using sound process; method implied by correct formula for mass of B obtained or greater accuracy (–0.12103..) or $0.5\ln 0.785$ accept 28 or greater accuracy accept 28 or greater accuracy	

Question			Answer	Marks	Guidance	
8	(i)		State $\cos 4\theta = 1 - 2\sin^2 2\theta$ State or clearly imply $\sin 2\theta = 2\sin\theta\cos\theta$ Obtain $1 - 8\sin^2\theta\cos^2\theta$	B1 B1 B1 **[3]**	possibly substituted in incorrect expression	
8	(ii)		Produce expression involving $\cos\frac{4}{24}\pi$ as only trigonometrical ratio Obtain $\frac{1}{8} - \frac{1}{16}\sqrt{3}$	M1 A1 **[2]**	or exact equiv (including, eg $\frac{1 - \frac{1}{2}\sqrt{3}}{8}$)	
8	(iii)		Use $2\cos^2 2\theta = 1 + \cos 4\theta$ Attempt to express in terms of $\cos 4\theta$ Obtain $\frac{2}{3} + \frac{4}{3}\cos 4\theta$ Substitute at least one of -1 and 1 for $\cos 4\theta$ in expression where $\cos 4\theta$ is only trigonometrical ratio Obtain 2 and $-\frac{2}{3}$	B1 M1 A1 M1 A1 **[5]**	or use $2\cos^2 2\theta = 2 - 8\sin^2\theta\cos^2\theta$ or unsimplified equiv or at least one of $\theta = \frac{1}{4}\pi$ and $\theta = 0$	

Question			Answer	Marks	Guidance	
9	(i)		Attempt differentiation to find x-coordinate of stationary point or attempt completion of square as far as $(x+...)^2$	M1	or equiv; first two marks of part **(i)** may be earned by work seen in part **(ii)**; $x=-2$ only stated earns M1A1	
			Obtain $x=-2$ or $(x+2)^2$	A1	first two marks of part **(i)** are implied by correct answer to translation in x-direction	
			State translation by 2 in negative x-direction	A1	or (clear) equiv; allow correct vector	
			State translation by 4 in negative y-direction	A1	or (clear) equiv; allow correct vector	
			State stretch parallel to y-axis, scale factor k	B1	or equiv at least mentioning y and k	
				[5]		
9	(ii)		State one of $y<4k,\ y\le 4k,\ y<-4k,\ y\le -4k$ $y>4k,\ y\ge 4k,\ y>-4k,\ y\ge -4k$	B1		
			State $y\ge -4k$	B1	allow alternative notation such as $f(x)\ge -4k$ or range $\ge -4k$	
				[2]		
9	(iii)		Attempt to relate y-value involving k at their stationary point to 20 or -20 or consider discriminant of $k(x^2+4x)=20$ or of $k(x^2+4x)=-20$	*M1		
			Obtain $k=5$	A1	earned unless there is clear evidence of error in working	
			State one root $x=-2$	B1		
			Attempt solution of $k(x^2+4x)=20$	M1	dep *M; for their value of k provided positive	
			Obtain $\frac{-4\pm\sqrt{32}}{2}$	A1ft	or (unsimplified) exact equivs; following their value of k	
			Obtain $-2\pm 2\sqrt{2}$ or $-2\pm\sqrt{8}$	A1	dependent on previous A1 A1ft marks being awarded	
				[6]		